Mark Scheme (Results)

Summer 2024

Pearson Edexcel GCE
In Further Mathematics (8FM0)
Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 80.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1(i)	$\alpha+\beta+\gamma=\frac{3}{2},\ \alpha\beta+\alpha\gamma+\beta\gamma=\frac{5}{2}$	B1	3.1a
	$\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\alpha\gamma+\beta\gamma)=\left(\frac{3}{2}\right)^2-2\left(\frac{5}{2}\right)=...$	M1	1.1b
	$=-\frac{11}{4}=-2.75$ cso	A1	1.1b
		(3)	
(ii)	$\alpha\beta\gamma=-\frac{7}{2}$ or $x=\frac{3}{w}$ used in the equation	B1	2.2a
	$\frac{3}{\alpha}+\frac{3}{\beta}+\frac{3}{\gamma}=\frac{3(\alpha\beta+\alpha\gamma+\beta\gamma)}{\alpha\beta\gamma}=\frac{3\left(\frac{5}{2}\right)}{\left(-\frac{7}{2}\right)}$ or $2\left(\frac{3}{w}\right)^3-3\left(\frac{3}{w}\right)^2+5\left(\frac{3}{w}\right)+7=0\Rightarrow 7w^3+15w^2-27w+54\{=0\}$ $\Rightarrow\ -\frac{'15'}{'7'}$	M1	1.1b
	$=-\frac{15}{7}$ cso	A1	1.1b
		(3)	
(iii)	$(5-\alpha)(5-\beta)(5-\gamma)=A\pm B(\alpha+\beta+\gamma)\pm C(\alpha\beta+\alpha\gamma+\beta\gamma)\pm(\alpha\beta\gamma)$ $=\{5^3-5^2(\alpha+\beta+\gamma)+5(\alpha\beta+\alpha\gamma+\beta\gamma)-\alpha\beta\gamma\}$ or $2(5-w)^3-3(5-w)^2+5(5-w)+7\{=0\}$ or $\mathrm{f}(x)=A(x-\alpha)(x-\beta)(x-\gamma)\Rightarrow A=2$	M1	3.1a
	$(5-\alpha)(5-\beta)(5-\gamma)=125-25\left(\frac{3}{2}\right)+5\left(\frac{5}{2}\right)+\frac{7}{2}$ or $(5-\alpha)(5-\beta)(5-\gamma)=-\left(\frac{2\times125-3\times25+25+7}{-2}\right)$ Or $-2w^3+27w^2-125w+207\{=0\}\Rightarrow-\frac{'207'}{'-2'}$ Or $\mathrm{f}(5)=2(5-\alpha)(5-\beta)(5-\gamma)$ $\Rightarrow(5-\alpha)(5-\beta)(5-\gamma)=\frac{\mathrm{f}(5)}{2}$	M1	1.1b

	$= \frac{207}{2} = 103.5$ cso	A1	1.1b
		(3)	
			(9 marks)

Notes

(i)
B1: Correct sum and pair sum, they may be seen anywhere in the candidates working.
M1: Uses a correct identity and substitutes in their sum and pair sum to find a value.
A1: Correct value following B1, if uses $\alpha+\beta+\gamma=-\frac{3}{2}$ this can score B0 M1 A0 cso

(ii)
B1: Correct value for the product (may be seen anywhere in the candidates working) or for using $x=\frac{3}{w}$ in the given equation.
M1: Uses a correct identity and substitutes in their pair sum and product to obtain a value or multiplies through by w^3 to identify at least the required terms and finds their new sum.
A1: Correct value from correct pair sum and product cso

(iii)
M1: Correct strategy for obtaining the required value by expanding, must reach an expression for the form $A \pm B(\alpha+\beta+\gamma) \pm C(\alpha\beta+\alpha\gamma+\beta\gamma) \pm (\alpha\beta\gamma)$ may not be factorised for example.

$A \pm B\alpha \pm B\beta \pm B\gamma \pm C\alpha\beta \pm C\alpha\gamma \pm C\beta\gamma \pm (\alpha\beta\gamma)$

or
Attempts the correct linear transformation of the given equation and expands.
or
Uses $\mathrm{f}(x)=A(x-\alpha)(x-\beta)(x-\gamma)$ to find a value for A

M1: Uses their sum, pair sum and product to obtain a value. Allow recovery from a sign slip as long as substituting into an expression of the form $A \pm B(\alpha+\beta+\gamma) \pm C(\alpha\beta+\alpha\gamma+\beta\gamma) \pm (\alpha\beta\gamma)$.
This would be A0 even if the correct answer is achieved.
or
Simplifies to obtain at least the required terms to find a value for the new product. Ignore the other terms whether correct or not.
Or
Uses $\frac{\mathrm{f}(5)}{2}$

A1: Correct value with no errors seen cso

Question	Scheme	Marks	AOs
2(a)	$\alpha = 210$	B1	1.1b
		(1)	
(b)	Require k×their '210' divisible by 360	M1	1.1b
	$k = 12$	A1	1.1b
		(2)	
(c)	$\{\mathbf{N} =\}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	B1	1.1b
		(1)	
(d)	$\mathbf{MA} = \begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} \sqrt{3}+2 \\ 1-2\sqrt{3} \\ 3 \end{pmatrix}$ $\mathbf{NMA} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \sqrt{3}+2 \\ 1-2\sqrt{3} \\ 3 \end{pmatrix} = \begin{pmatrix} \sqrt{3}+2 \\ 2\sqrt{3}-1 \\ 3 \end{pmatrix}$ * Or $\mathbf{NM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $\mathbf{NMA} = \begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} \sqrt{3}+2 \\ 2\sqrt{3}-1 \\ 3 \end{pmatrix}$ *	M1	1.1a
	i.e. $B\left(2+\sqrt{3},\ 2\sqrt{3}-1,\ 3\right)$ *	A1*	1.1b
		(2)	

(e)	$AB^2 = OA^2 + OB^2 - 2OA.OB\cos AOB$ $\Rightarrow \left(4+\sqrt{3}\right)^2 + \left(5-2\sqrt{3}\right)^2 = 29+29-2\sqrt{29}.\sqrt{29}\cos AOB$ or $OA.OB = \begin{pmatrix}-2\\4\\3\end{pmatrix} \bullet \begin{pmatrix}2+\sqrt{3}\\2\sqrt{3}-1\\3\end{pmatrix} = 1+6\sqrt{3} = \sqrt{29}\sqrt{29}\cos AOB$	M1	3.1a
	$AOB = 66.9°$ *	A1*	1.1b
		(2)	
(f)	Area $AOB = \frac{1}{2}\sqrt{29}\sqrt{29}\sin 66.9°$ You may see this outside spec from candidates studying 8FM0 21 $\frac{1}{2}\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ -2 & 4 & 3\\ 2+\sqrt{3} & 2\sqrt{3}-1 & 3\end{vmatrix}$ $= \frac{1}{2}\left\|\left[12-3\left(2\sqrt{3}-1\right)\right]\mathbf{i} - \left[-6-3\left(2+\sqrt{3}\right)\right]\mathbf{j} + \left[-2\left(2\sqrt{3}-1\right)-4\left(2+\sqrt{3}\right)\right]\mathbf{k}\right\|$ $= \frac{1}{2}\left\|\left(15-6\sqrt{3}\right)\mathbf{i} + \left(12+3\sqrt{3}\right)\mathbf{j} + \left(-6-8\sqrt{3}\right)\mathbf{k}\right\|$ $= \frac{1}{2}\sqrt{\left(15-6\sqrt{3}\right)^2 + \left(12+3\sqrt{3}\right)^2 + \left(-6-8\sqrt{3}\right)^2}$	M1	1.1b
	= 13.3 cao	A1	1.1b
		(2)	
			(10 marks)

Notes

(a)
B1: Correct value, check within the question. If more than one value is stated the correct value must clearly be selected.

(b)
M1: Uses their answer from part (a) to determine a value for k so that $k\times$their 210 is divisible by 360. If their answer to part (a) is in radians $k \times$ their $\frac{7\pi}{6}$ is divisible by 2π.

A1: Correct value must be using an angle of 210. There may be no working but allow M1 A1 for $k = 12$ following an answer of 210 or $\frac{7\pi}{6}$ in part (a)

Note: an angle of 30, 150, 330 also gives *k* = 12 but is M1A0

(c)
B1: Correct matrix

(d)
M1: Complete method to find the coordinates of B. Look for at least two correct terms for each multiplication, follow through when multiplying by **N.**
Alternatively finds **NM** (not **MN**), look for 4 correct non zero terms if no method is shown and then multiplies to find the coordinates of B.

A1*: Correct coordinates (condone vector) Must have been working with exact values throughout. If working in decimals M1 A0

It is insufficient to just write $\mathbf{NM}\begin{pmatrix}-2\\4\\3\end{pmatrix}=\begin{pmatrix}\sqrt{3}+2\\2\sqrt{3}-1\\3\end{pmatrix}$ there must be some evidence of matrix multiplication seen.

(e)

M1: Identifies and applies an appropriate strategy to find the required angle e.g. cosine rule or scalar product. Note: $AB^2=56-12\sqrt{3}$ and $AB=3\sqrt{6}-\sqrt{2}=5.93...$

A1*: Correct value from correct equation

(f)

M1: Uses the given angle with $\frac{1}{2}ab\sin C$ with appropriate *a*, *b* and *C*

Outside spec: uses the cross product $\frac{1}{2}\left|\overrightarrow{OA}\times\overrightarrow{OB}\right|$

A1: Correct area to 3 significant figures

Question	Scheme	Marks	AOs
3(a)	$\sum_{r=1}^{n} r^2(r+1) = \sum_{r=1}^{n} r^3 + r^2 = \frac{1}{4}n^2(n+1)^2 + \frac{1}{6}n(n+1)(2n+1)$	M1 A1	1.1b 1.1b
	$= \frac{1}{12}n(n+1)\left[3n(n+1)+2(2n+1)\right]$	dM1	1.1b
	$= \frac{1}{12}n(n+1)\left[3n^2+7n+2\right] = \frac{1}{12}n(n+1)(n+2)(3n+1)$ cso	A1	2.1
		(4)	
(b)	$\sum_{r=k+1}^{3k} r^2(r+1) = \frac{1}{12}(3k)(3k+1)(3k+2)(9k+1) - \frac{1}{12}(k)(k+1)(k+2)(3k+1)$	M1	3.1a
	$= \frac{1}{12}k(3k+1)\left[3(3k+2)(9k+1)-(k+1)(k+2)\right]$ or $= \frac{1}{3}k(3k+1)\left[\frac{3}{4}(3k+2)(9k+1)-\frac{1}{4}(k+1)(k+2)\right]$	M1	1.1b
	$= \frac{1}{12}k(3k+1)\left[80k^2+60k+4\right]$ $= \frac{1}{3}k(3k+1)\left(20k^2+15k+1\right)$ cso	A1	1.1b
		(3)	
(c)	$\frac{25}{3}k(3k+1)\left(20k^2+15k+1\right) = 192k^3(3k+1)$ either $\Rightarrow 25\left(20k^2+15k+1\right) = 576k^2 \Rightarrow 76k^2-375k-25=0$ Or $\Rightarrow 25k\left(20k^2+15k+1\right) = 576k^3 \Rightarrow 76k^3-375k^2-25k=0$ Or $\Rightarrow 25(3k+1)\left(20k^2+15k+1\right) = 576k^2(3k+1) \Rightarrow 228k^3-1049k^2-450k-25=0$ Or $-76k^4+\frac{1049}{3}k^3+150k^2+\frac{25}{3}k=0$	M1	1.1b
	$76k^2-375k-25=0 \Rightarrow k=...$ $76k^3-375k^2-25k=0 \Rightarrow k=...$ $-76k^4+\frac{1049}{3}k^3+150k^2+\frac{25}{3}k=0 \Rightarrow k=...$	M1	1.1b
	$k=5$ (only)	A1	2.3
		(3)	
		(10 marks)	

Notes

(a)

M1: Substitutes at least one of the standard formulae into their expanded expression

A1: Fully correct expression

dM1: Attempts to factorise $\frac{1}{12}n(n+1)$ or $\frac{1}{3}n(n+1)$ having used at least one standard formula correctly at any stage. Dependent on the first M mark.

If they show no method for factorising (use a calculator) they can go from

$3n^3+10n^2+9n+2=(n+1)(n+2)(3n+1)$

A1: Obtains the correct expression or the correct values of *a* and *b*, with no errors seen

(b)

M1: Uses the result from part (a) and adopts a correct strategy by attempting

$\sum_{r=1}^{3k} r^2(r+1)-\sum_{r=1}^{k} r^2(r+1)$

M1: Factorises out at least $k(3k+1)$ at any stage, could be done by inspection

A1: Obtains the correct expression with no errors seen.

Note If a candidate does not use part (a) but restarts they can still score marks for the same reasons. If unsure please send to review

(c)

M1: Uses the given equation, substitutes their answer from part (b) and simplifies to either reach

- $Ak^4+Bk^3+Ck^2+Dk\{=0\}$
- $k(Ak^3+Bk^2+Ck+D)\{=0\}$ or $(3k+1)(Ak^3+Bk^2+Ck)\{=0\}$
- $Ak^3+Bk^2+Ck\{=0\}$
- a 3TQ or $k(3k+1)(Ak^2+Bk+C)\{=0\}$

this can be implied by a correct value for *k* if left unsimplified

M1: Solves their equation as long as solving their $(b)=192k^3(3k+1)$ to find a non zero value for *k*, including by calculator. You may need to check this.

A1: Selects the appropriate correct answer of $k = 5$. Any other solutions must be clearly rejected.

Note: Correct answer with no working is M0M0A0

Question	Scheme	Marks	AOs
4(a)	$-4k+2+20-21+7k$ or $3+3k-2$ or $7k-4-19+24-4k$ or $3k+1$	M1	1.1b
	$\{1+3k=3k+c\}$ $\Rightarrow c=1$	A1	1.1b
		(2)	
(b)	$3k+1=0 \Rightarrow k=...$ Or Attempts the determinant and sets = 0 leading to a value for k	M1	1.1b
	$\Rightarrow k=-\frac{1}{3}$	A1ft	1.1b
		(2)	
(c)	$\{\mathbf{A}^{-1}\}=\frac{1}{3k+1}\begin{pmatrix}4k-2 & 1 & 7k-4\\ -10 & 3 & -19\\ 3-k & -1 & 6-k\end{pmatrix}$	B1ft	2.2a
		(1)	
(d)	$\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\frac{1}{3k+1}\begin{pmatrix}4k-2 & 1 & 7k-4\\ -10 & 3 & -19\\ 3-k & -1 & 6-k\end{pmatrix}\begin{pmatrix}10\\ 3\\ 1\end{pmatrix}=...$	M1	1.2
	$\left(\frac{47k-21}{3k+1}, -\frac{110}{3k+1}, \frac{33-11k}{3k+1}\right)$	A1 A1	1.1b 1.1b
		(3)	
			(8 marks)

Notes

(a)
M1: Calculates one of the elements of the leading diagonal of **AB**, condone sign slips
A1: Sets diagonal = $3k+c$ and deduces the correct value for c. Award for sight of $3k+1$
(b)
M1: Attempts to solve $3k+\text{"1"}=0$ or attempts the determinant, condone sign slips in the minors, and sets = 0 leading to a value for k

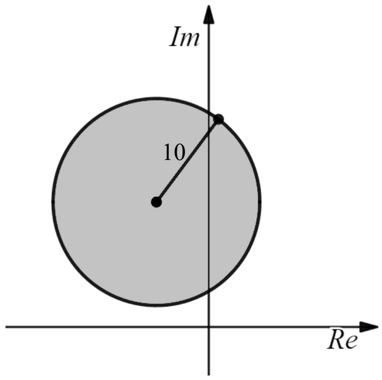
A1ft: Correct value or follow through their value for c so allow for $k=-\frac{c}{3}$

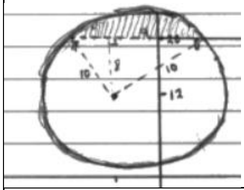
(c)

B1ft: Deduces the correct inverse matrix. Follow through their c so allow for $\frac{1}{3k+c}\mathbf{B}$ or if found determinant $\frac{1}{\text{their det}}\mathbf{B}$

(d)
M1: Complete method to find the values of x, y and z using their inverse matrix

A1: At least one correct coordinate simplified or unsimplified. A1: All coordinates correct and simplified. Condone as a column vector. Does not need to be written as a coordinate. SC If candidate writes $\frac{1}{3k+1}\begin{pmatrix}4k-2 & 1 & 7k-4\\ -10 & 3 & -19\\ 3-k & -1 & 6-k\end{pmatrix}\begin{pmatrix}10\\ 3\\ 1\end{pmatrix}$ but ends up with **at least one of** $x=47k-21,\ y=-110,\ z=33-11k$ scores M1 A1 A0

Question	Scheme	Marks	AOs
5(a)(i)	(−5, 12) or −5 + 12i	B1	1.1b
(ii)	$r = 10$	B1	1.1b
		(2)	
(b)		B1ft	1.1b
		(1)	
(c)	$OC = \sqrt{5^2 + 12^2}$	M1	1.1b
	$\lvert z \rvert_{\max} = \sqrt{5^2 + 12^2} + 10$	M1	3.1a
	$= 23$	A1	1.1b
		(3)	
	Alternative $y = -\frac{12}{5}x$ and $(x+5)^2 + (y-12)^2 = 10^2$ $(x+5)^2 + \left(-\frac{12}{5}x - 12\right)^2 = 10^2 \Rightarrow x = ...$ Or $\tan\theta = \frac{5}{12} \Rightarrow \theta = ...\{22.61..\}\quad x = -5 - 10\sin\theta = ...$	M1	1.1b
	$x = '-\frac{115}{13}' \Rightarrow y = ...\left\{\frac{276}{13}\right\}$ $\lvert z \rvert_{\max} = \sqrt{\left(-\frac{115}{13}\right)^2 + \left(\frac{276}{13}\right)^2}$ Or $x = '-\frac{15}{13}' \Rightarrow y = ...\left\{\frac{36}{13}\right\}$ $\lvert z \rvert_{\max} = \sqrt{\left(-\frac{15}{13}\right)^2 + \left(\frac{36}{13}\right)^2} + 2 \times 10$	M1	3.1a
	$= 23$	A1	1.1b

		(3)	
(d)	$\{z: 0 \leqslant \arg(z+5-20\text{i}) \leqslant \pi\} \Rightarrow y = 20$ $\Rightarrow (x+5)^2 + 8^2 = 100 \Rightarrow x = \ldots$ **AND finds an angle** $\cos\theta = \dfrac{10^2+10^2-12^2}{2\times10\times10} = 0.28$ Or $a^2 = 10^2 - 8^2 \Rightarrow a = \ldots\{6\}\ \sin\left(\dfrac{1}{2}\theta\right) = \dfrac{6}{10}$ Or $\cos\left(\dfrac{1}{2}\theta\right) = \dfrac{8}{10}$ 	M1	3.1a
	$\theta = 1.287\ldots$or 73.7° or $\dfrac{1}{2}\theta = 0.6435\ldots$or 36.9°	A1	1.1b
	Area $= \dfrac{1}{2}\times10^2\times\theta - \dfrac{1}{2}\times12\times8$ angle in radians Area $= \pi\times10^2\times\dfrac{\theta}{360} - \dfrac{1}{2}\times12\times8$ angle in degrees or Area $= \dfrac{1}{2}\times10^2\times\theta - \dfrac{1}{2}\times10\times10\times\sin\theta$ angle in radians Area $= \pi\times10^2\times\dfrac{\theta}{360} - \dfrac{1}{2}\times10\times10\times\sin\theta$ angle in degrees Or Area $= 2\left[\dfrac{1}{2}\times10^2\times\theta - \dfrac{1}{2}\times8\times6\right]$	M1	3.1a
	= awrt 16.4	A1	1.1b
		(4)	
			(10 marks)

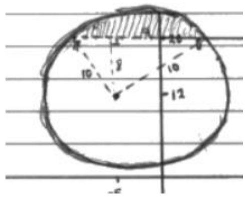
Notes

(a)(i)
B1: Correct centre, condone $(-5, 12\text{i})$

(a)(ii)
B1: Correct radius

(b)
B1ft: A circle drawn with the inside shaded. Follow through their centre and radius.
The centre must be in the correct quadrant and intercept the axes as appropriate.
If they have the correct centre and radius then the centre must be in the second quadrant and the circle must **only** intercept the imaginary-axis. If diagram is correct consider this a restart B1.

(c)
M1: Calculates the distance from O to the centre of their circle.
M1: Fully correct strategy for the maximum. E.g. Finds distance from O to centre of their circle and adds their radius.
A1: Correct answer.
Correct answer with no working and following a correct centre and radius scores M1M1A1
Alternative
M1: Finds the equation of the line from the origin to centre and the Cartesian equation of the circle. Solves simultaneously to find the x values (or y) where the line intersects the circle.
M1: Selects the x coordinate to give the largest distance, find the corresponding y value and then the distance from the origin.
Selects the x coordinate to give the smallest distance, find the corresponding y value and then adds on 2 times the radius.
A1: Correct answer
(d)
M1: Recognises that $\{z: 0 \leqslant \arg(z+5-20\mathrm{i}) \leqslant \pi\}$ represents the line $y = 20$ and uses this with the circle in an attempt to find the angle or half angle at the centre.



A1: Correct value for the angle or half angle at the centre.
M1: Fully correct strategy for the area of the segment using their values.
A1: Awrt 16.4
Note: finding the area of the major segment using $100\pi - 16.4 = \ldots$ scores M1A1M1A0

Question	Scheme	Marks	AOs
6(a)	Direction: $\pm\left(3\mathbf{i}+4\mathbf{j}-2\mathbf{k}-(-2\mathbf{i}-8\mathbf{j}-3\mathbf{k})\right)$	M1	1.1b
	e.g. $\mathbf{r}=3\mathbf{i}+4\mathbf{j}-2\mathbf{k}+\lambda(5\mathbf{i}+12\mathbf{j}+\mathbf{k})$ $\mathbf{r}=-2\mathbf{i}-8\mathbf{j}-3\mathbf{k}+\lambda(5\mathbf{i}+12\mathbf{j}+\mathbf{k})$	A1	2.5
		(2)	
(b)	$z=0\Rightarrow -2+\lambda=0\Rightarrow\lambda=2\Rightarrow C=...$	M1	1.1b
	$\lambda=2\Rightarrow C$ is $(13,\ 28,\ 0)$	A1	1.1b
		(2)	
(c)	$(5\mathbf{i}+12\mathbf{j}+\mathbf{k})\cdot(2\mathbf{i}+4\mathbf{j}-2\mathbf{k})=10+48-2$	M1	3.1b
	$56=\sqrt{5^2+12^2+1^2}\sqrt{2^2+4^2+2^2}\cos\theta\Rightarrow\cos\theta=\dfrac{56}{\sqrt{170}\sqrt{24}}$	M1	1.1b
	$\Rightarrow\theta=\text{awrt } 28.8°$	A1	1.1b
		(3)	
(d)	$\mathbf{P}_1-\mathbf{P}_2=3\mathbf{i}+4\mathbf{j}-2\mathbf{k}+\lambda(5\mathbf{i}+12\mathbf{j}+\mathbf{k})-(\mathbf{i}+3\mathbf{j}-\mathbf{k}+\mu(2\mathbf{i}+4\mathbf{j}-2\mathbf{k}))$ or $\mathbf{P}_1-\mathbf{P}_2=-2\mathbf{i}-8\mathbf{j}-3\mathbf{k}+\lambda(5\mathbf{i}+12\mathbf{j}+\mathbf{k})-(\mathbf{i}+3\mathbf{j}-\mathbf{k}+\mu(2\mathbf{i}+4\mathbf{j}-2\mathbf{k}))$	M1	3.4
	$((5\lambda-2\mu+2)\mathbf{i}+(12\lambda-4\mu+1)\mathbf{j}+(\lambda+2\mu-1)\mathbf{k})\cdot(5\mathbf{i}+12\mathbf{j}+\mathbf{k})=0$ $\begin{pmatrix}2+5\lambda-2\mu\\1+12\lambda-4\mu\\-1+\lambda+2\mu\end{pmatrix}\bullet\begin{pmatrix}5\\12\\1\end{pmatrix}$ $\Rightarrow 170\lambda-56\mu=-21$ **AND** $((5\lambda-2\mu+2)\mathbf{i}+(12\lambda-4\mu+1)\mathbf{j}+(\lambda+2\mu-1)\mathbf{k})\cdot(2\mathbf{i}+4\mathbf{j}-2\mathbf{k})=0$ $\begin{pmatrix}2+5\lambda-2\mu\\1+12\lambda-4\mu\\-1+\lambda+2\mu\end{pmatrix}\bullet\begin{pmatrix}2\\4\\-2\end{pmatrix}$ $\Rightarrow 56\lambda-24\mu=-10$	M1	3.1b
	$170\lambda-56\mu=-21,\ \ 56\lambda-24\mu=-10\Rightarrow\lambda=...\left(\dfrac{7}{118}\right),\ \mu=...\left(\dfrac{131}{236}\right)$ If using $\mathbf{r}=-2\mathbf{i}-8\mathbf{j}-3\mathbf{k}+\lambda(5\mathbf{i}+12\mathbf{j}+\mathbf{k})$ this leads to parameters of $\lambda=...\left(\dfrac{125}{118}\right),\ \mu=...\left(\dfrac{131}{236}\right)$ **Either** $\mathbf{P}_1-\mathbf{P}_2=...\left(\dfrac{70}{59}\mathbf{i}-\dfrac{30}{59}\mathbf{j}+\dfrac{10}{59}\mathbf{k}\right)$ or $\lvert\mathbf{P}_1-\mathbf{P}_2\rvert=...$	M1	3.4

	$\lvert \mathbf{P}_1 - \mathbf{P}_2 \rvert = \sqrt{\left(\frac{70}{59}\right)^2 + \left(\frac{30}{59}\right)^2 + \left(\frac{10}{59}\right)^2} = \ldots$	dM1	1.1b
	Awrt 1.3{0} **m** or $\frac{10\sqrt{59}}{59}$ **m units required**	A1	3.2a
		(5)	
	Alternative 1 $\mathbf{P}_1 - \mathbf{P}_2 = 3\mathbf{i} + 4\mathbf{j} - 2\mathbf{k} + \lambda(5\mathbf{i} + 12\mathbf{j} + \mathbf{k}) - (\mathbf{i} + 3\mathbf{j} - \mathbf{k} + \mu(2\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}))$	M1	3.4
	Finds $\begin{pmatrix} 5 \\ 12 \\ 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix} = \ldots\{-28\mathbf{i} + 12\mathbf{j} - 4\mathbf{k}\}$ $\begin{pmatrix} 2 + 5\lambda - 2\mu \\ 1 + 12\lambda - 4\mu \\ -1 + \lambda + 2\mu \end{pmatrix} = M \begin{pmatrix} -28 \\ 12 \\ -4 \end{pmatrix}$ Or $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \bullet \begin{pmatrix} 5 \\ 12 \\ 1 \end{pmatrix} = 5x + 12y + z = 0$ And $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix} = 2x + 4y - 2z = 0$ Leding to a vector e.g. $\begin{pmatrix} 7 \\ -3 \\ 1 \end{pmatrix}$	M1	3.1b
	$\begin{matrix} 2 + 5\lambda - 2\mu = -28M \\ 1 + 12\lambda - 4\mu = 12M \\ -1 + \lambda + 2\mu = -4M \end{matrix} \Rightarrow \lambda = \ldots\left(\frac{7}{118}\right),\ \mu = \ldots\left(\frac{131}{236}\right), \left\{M = -\frac{5}{118}\right\}$ $\mathbf{P}_1 - \mathbf{P}_2 = \ldots\left(\frac{70}{59}\mathbf{i} - \frac{30}{59}\mathbf{j} + \frac{10}{59}\mathbf{k}\right)$	M1	3.4
	$\lvert \mathbf{P}_1 - \mathbf{P}_2 \rvert = \sqrt{\left(\frac{70}{59}\right)^2 + \left(\frac{30}{59}\right)^2 + \left(\frac{10}{59}\right)^2}$	dM1	1.1b
	Awrt 1.3{0} **m** or $\frac{10\sqrt{59}}{59}$ **m units required**	A1	3.2a
		(5)	
	Alternative 2 outside the spec	M1	3.4

	$\mathbf{r}=\begin{pmatrix}3\\4\\-2\end{pmatrix}+\lambda\begin{pmatrix}5\\12\\1\end{pmatrix}$ and $\mathbf{r}=\begin{pmatrix}1\\3\\-1\end{pmatrix}+\lambda\begin{pmatrix}2\\4\\-2\end{pmatrix}$ $\begin{pmatrix}3\\4\\-2\end{pmatrix}-\begin{pmatrix}1\\3\\-1\end{pmatrix}=\begin{pmatrix}2\\1\\-1\end{pmatrix}$		
	$\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\5&12&1\\2&4&-2\end{vmatrix}=\mathbf{i}(-24-4)-\mathbf{j}(-10-2)+\mathbf{k}(20-24)$ $=-28\mathbf{i}+12\mathbf{j}-4\mathbf{k}$ And $\begin{pmatrix}2\\1\\-1\end{pmatrix}\bullet\begin{pmatrix}-28\\12\\-4\end{pmatrix}=-56+12+4=...\{-40\}$	M1	3.4
	$\lvert-28\mathbf{i}+12\mathbf{j}-4\mathbf{k}\rvert=\sqrt{(-28)^2+12^2+(-4)^2}=...\{\sqrt{944}\}$	dM1	3.1b
	$\lvert\mathbf{P}_1-\mathbf{P}_2\rvert=\left\lvert\frac{-40}{\sqrt{944}}\right\rvert$	M1	1.1b
	Awrt 1.3{0} **m** or $\frac{10\sqrt{59}}{59}$ **m units required**	A1	3.2a
		(5)	
			(12 marks)

Notes

(a)
M1: Subtracts the given coordinates either way round, 2 correct values implies method, and uses as their direction vector.
A1: For a correct equation. Allow any equivalent correct equations but must use the correct notation, starts with **r** = …, can be using column vectors
(b)
M1: Uses $z = 0$ in their P_1 equation to find a value for their parameter and uses this to find the other coordinates.
A1: Deduces the correct coordinates, condone written as a vector.
(c)
M1: Realises the scalar product between their direction from part (a) and the direction vector extracted from the P_2 equation is required and calculates its value
M1: Completes the method and at least reaches a value for cosine of the angle. Must be attempting to use the direction vectors
A1: Correct acute angle
(d)
M1: Uses the model to form a general vector connecting the two pipes, condone sign slips if the intention is clear. Condone use of the same parameter for this mark. They will be unable to score any more marks
M1: Recognises that the scalar product between the general vector **and** the directions of the lines = 0 and uses this to form 2 simultaneous equations in terms of their parameters. Follow through on their line from part (a)

M1: Attempts to solve the simultaneous equations, writing a value for each parameter is sufficient.
Then uses the values of their parameters in the model and **either**

- shows the shortest vector connecting the 2 lines
- proceeds to a distance with no incorrect working seen.

dM1: Uses their values of their parameters to find the vector which if not correct must be stated and then finds the modulus of their shortest vector to find the shortest distance. Maybe implied by a correct answer. Dependent on previous method mark.
A1: Awrt 1.3(0) **m**
Alternative 1
M1: Uses the model to form a general vector connecting the two pipes, condone sign slips if the intention is clear.
M1: Finds the normal vector to the direction vectors by any method and sets the general equation of the line equal to a multiple of the normal vector.
M1: Attempts to solve the simultaneous equations, writing a value for each parameter is sufficient.
Then uses the values of their parameters in the model and **either**

- shows the shortest vector connecting the 2 lines
- proceeds to a distance with no incorrect working seen.

dM1: Uses their values of their parameters to find the vector which if not correct must be stated and then finds the modulus of their shortest vector to find the shortest distance. Maybe implied by a correct answer. Dependent on previous method mark.
A1: Awrt 1.3(0) **m**

Alternative 2 outside spec
M1: Using their equations to find the vector between the coordinates $(\mathbf{a}-\mathbf{c})$
M1: Find the cross product of their directions $(\mathbf{b}\times\mathbf{d})$ and finds the dot product of their $(\mathbf{a}-\mathbf{c})\bullet(\mathbf{b}\times\mathbf{d})$
M1: Finds their $|\mathbf{b}\times\mathbf{d}|$
M1: Uses $\left|\dfrac{(\mathbf{a}-\mathbf{c})\bullet(\mathbf{b}\times\mathbf{d})}{|\mathbf{b}\times\mathbf{d}|}\right|$
A1: Awrt 1.3(0) **m.**

Question	Scheme	Marks	AOs
7(i)	$n=1,\quad lhs=\frac{1}{1(2)}=\frac{1}{2}\quad rhs=\frac{1}{1+1}=\frac{1}{2}$ So the result is true for n = 1	B1	2.2a
	Assume true for $n = k$ so $\sum_{r=1}^{k}\frac{1}{r(r+1)}=\frac{k}{k+1}$	M1	2.4
	$\sum_{r=1}^{k+1}\frac{1}{r(r+1)}=\frac{k}{k+1}+\frac{1}{(k+1)(k+2)}=\frac{k(k+2)}{(k+1)(k+2)}+\frac{1}{(k+1)(k+2)}$	M1	2.1
	$=\frac{k(k+2)+1}{(k+1)(k+2)}=\frac{k^2+2k+1}{(k+1)(k+2)}=\frac{(k+1)^2}{(k+1)(k+2)}=\frac{\underline{k+1}}{\underline{k+1}+1}$ cso	A1	1.1b
	If true for *n* = *k* then it has been shown **true for *n* = *k* + 1** and as it is **true for *n* = 1,** the statement is **true for all positive integers *n*.**	A1	2.4
		(5)	
7(ii)	**Way 1: f (*k* + 1)** $n=1,\quad 3^{2n+4}-2^{2n}=3^6-2^2=729-4=725$ So the result is true for n = 1 as 725 is divisible by 5	B1	2.2a
	Assume true for $n = k$ so is $3^{2k+4}-2^{2k}$ divisible by 5	M1	2.4
	$\text{f}(k+1)=3^{2(k+1)+4}-2^{2(k+1)}=3^{2k+6}-2^{2k+2}$ Look for $A\times3^{2k+4}-A\times2^{2k}+B\times2^{2k}$ or $A\times3^{2k+4}-A\times2^{2k}+B\times3^{2k+4}$ $9\times3^{2k+4}-9\times2^{2k}+5\times2^{2k}$ or $4\times3^{2k+4}-4\times2^{2k}+5\times3^{2k+4}$	M1	2.1
	$=9\text{f}(k)+5\times2^{2k}$ or $=4\text{f}(k)+5\times3^{2k+4}$	A1	1.1b
	If true for *n* = *k* then it has been shown **true for *n* = *k* + 1** and as it is **true for *n* = 1,** the statement is **true for all positive integers *n*.**	A1	2.4
		(5)	
	Way 2: f (*k* + 1)- f (*k*) $n=1,\quad 3^{2n+4}-2^{2n}=3^6-2^2=729-4=725$ So the result is true for n = 1 as 725 is divisible by 5	B1	2.2a
	Assume true for $n = k$ so $3^{2k+4}-2^{2k}$ is divisible by 5	M1	2.4
	$\text{f}(k+1)-\text{f}(k)=3^{2(k+1)+4}-2^{2(k+1)}-3^{2k+4}+2^{2k}$ Look for $A\times3^{2k+4}-A\times2^{2k}+B\times2^{2k}$ or $A\times3^{2k+4}-A\times2^{2k}+B\times3^{2k+4}$ $=8\times3^{2k+4}-8\times2^{2k}+5\times2^{2k}$ or $3\times3^{2k+4}-3\times2^{2k}+5\times3^{2k+4}$	M1	2.1
	$\text{f}(k+1)=9\text{f}(k)+5\times2^{2k}$	A1	1.1b

	or $\mathrm{f}(k+1)=4\mathrm{f}(k)+5\times 3^{2k+4}$		
	If true for *n* = *k* then it has been shown **true for *n* = *k* + 1** and as it is **true for *n* = 1,** the statement is **true for all positive integers *n*.**	A1	2.4
		(5)	
	Way 3: $\mathbf{f}(\boldsymbol{k})=5\boldsymbol{M}$ $n=1,\quad 3^{2n+4}-2^{2n}=3^6-2^2=729-4=725$ So the result is true for $n = 1$ as 725 is divisible by 5	B1	2.2a
	Assume true for $n = k$ so $3^{2k+4}-2^{2k}=5M$ is divisible by 5	M1	2.4
	$\mathrm{f}(k+1)=3^{2(k+1)+4}-2^{2(k+1)}\left(=3^{2k+6}-2^{2k+2}\right)$ $\mathrm{f}(k+1)=3^2\times 3^{2k+4}-2^2\times 2^{2k}=3^2\times\left(5M+2^{2k}\right)-2^2\times 2^{2k}$ OR $\mathrm{f}(k+1)=3^2\times 3^{2k+4}-2^2\times 2^{2k}=3^2\times 3^{2k+4}-2^2\times\left(3^{2k+4}-5M\right)$	M1	2.1
	$\mathrm{f}(k+1)=45M+5\times 2^{2k}$ OR $\mathrm{f}(k+1)=5\times 3^{2k+4}+20M$ o.e.	A1	1.1b
	If true for *n* = *k* then it has been shown **true for *n* = *k* + 1** and as it is **true for *n* = 1,** the statement is **true for all positive integers *n*.**	A1	2.4
		(5)	
	Way 4: $\mathbf{f}(\boldsymbol{k}+1)-\boldsymbol{m}\,\mathbf{f}(\boldsymbol{k})$ $n=1,\quad 3^{2n+4}-2^{2n}=3^6-2^2=729-4=725$ So the result is true for $n = 1$ as 725 is divisible by 5	B1	2.2a
	Assume true for $n = k$ so $3^{2k+4}-2^{2k}$ is divisible by 5	M1	2.4
	$\mathrm{f}(k+1)-m\mathrm{f}(k)=3^{2k+6}-2^{2k+2}-m\left(3^{2k+4}-2^{2k}\right)$ $3^2\times 3^{2k+4}-2^2\times 2^{2k}-m\times 3^{2k+4}+m\times 2^{2k}$ $(9-m)\times 3^{2k+4}-4\times 2^{2k}+m\times 2^{2k}$ $(9-m)\times\left(3^{2k+4}-2^{2k}\right)+5\times 2^{2k}$	M1	2.1
	$\mathrm{f}(k+1)=(9-m)\times\mathrm{f}(k)+5\times 2^{2k}+m\mathrm{f}(k)$ $\mathrm{f}(k+1)=(9-m)\times\left(3^{2k+4}-2^{2k}\right)+5\times 2^{2k}+m\mathrm{f}(k)$ Note if $m=4$ leads to $5\times\left(3^{2k+4}\right)$	A1	1.1b
	If true for *n* = *k* then it has been shown **true for *n* = *k* + 1** and as it is **true for *n* = 1,** the statement is **true for all positive integers *n*.**	A1	2.4
		(5)	
			(10 marks)

Notes

(i) Must be using induction to score any marks

B1: Shows that the result holds for $n = 1$. Minimum LHS = $\frac{1}{1(2)}$ RHS $= \frac{1}{1+1}$

M1: Makes a statement that assumes the result is true for some value of n, say k

M1: Attempts to add the next term and makes progress by attempting a common denominator.

A1: Achieves a correct expression in terms of $k + 1$, with no errors seen cso. Alternatively finds $\sum_{r=1}^{k+1} \frac{1}{r(r+1)} = \frac{k+1}{k+2}$ and reaches the same expression using induction.

A1: Correct conclusion. This mark is dependent on all previous marks. It is gained by conveying the ideas of **all four bold** points either at the end of their solution or as a narrative in their solution. **Not dependent on B1 as long as attempted**

(ii)

Way 1: f (k + 1)

B1: Shows that the result holds for $n = 1$

M1: Makes a statement that assumes the result is true for some value of n, say k

M1: Attempts f (k + 1) and attempts to express in terms of f (k)

A1: Achieves a correct expression in terms of f (k)

A1: Correct conclusion. This mark is dependent on all previous marks. It is gained by conveying the ideas of **all four bold** points either at the end of their solution or as a narrative in their solution. **Not dependent on B1 as long as attempted**

Way 2: f (k + 1)- f (k)

B1: Shows that the result holds for $n = 1$

M1: Makes a statement that assumes the result is true for some value of n, say k

M1: Attempts f (k + 1) – f (k) and attempts to express in terms of f (k)

A1: Achieves a correct expression for f (k + 1) in terms of f (k)

A1: Correct conclusion. This mark is dependent on all previous marks. It is gained by conveying the ideas of **all four bold** points either at the end of their solution or as a narrative in their solution. **Not dependent on B1 as long as attempted**

Way 3: f (k)= 5M

B1: Shows that the result holds for $n = 1$

M1: Makes a statement that assumes the result is true for some value of n, say k

M1: Attempts f(k +1) and writes in terms of 5M.

A1: Achieves a correct expression for f (k + 1) in terms of M and 2^{2k} or M and 3^{2k+4}

A1: Correct conclusion. This mark is dependent on all previous marks. It is gained by conveying the ideas of **all four bold** points either at the end of their solution or as a narrative in their solution. **Not dependent on B1 as long as attempted**

Way 4: f (k + 1)- mf (k)

B1: Shows that the result holds for $n = 1$

M1: Makes a statement that assumes the result is true for some value of n, say k

M1: Attempts f (k + 1) – m f (k) and attempts to express in terms of f (k)

A1: Achieves a correct expression for f (k + 1) in terms of f (k)

A1: Correct conclusion. This mark is dependent on all previous marks. It is gained by conveying the ideas of **all four bold** points either at the end of their solution or as a narrative in their solution. **Not dependent on B1 as long as attempted**

Note conclusion may be in terms of $\mathrm{f}(1), \mathrm{f}(k), \mathrm{f}(k+1)$			
Question	**Scheme**	**Marks**	**AOs**
8(a)	$(0.4,\ 4) \Rightarrow 0.4 = k \times 4^2 + \sqrt{4} \Rightarrow k = \ldots$	M1	3.3
	$k = -0.1$	A1	1.1b
		(2)	
(b)	Cylinder volume = $\pi \times 0.4^2 \times 0.5 = 0.08\pi = \frac{2}{25}\pi$	B1	3.4
	Volume generated by curve = $\pi \int x^2 \, \mathrm{d}y$ $\pi \int \left(\sqrt{y} + ky^2\right)^2 \{\mathrm{d}y\} = \pi \int \left(\sqrt{y} - 0.1y^2\right)^2 \{\mathrm{d}y\}$	M1	3.1b
	$= \{\pi\} \int \left(y + 2ky^{\frac{5}{2}} + k^2 y^4 \right) \{\mathrm{d}y\}$ $= \{\pi\} \int \left(y - 0.2y^{\frac{5}{2}} + 0.01y^4 \right) \{\mathrm{d}y\}$	A1ft	1.1b
	$= \{\pi\} \int_0 \left(y - 0.2y^{\frac{5}{2}} + 0.01y^4 \right) \{\mathrm{d}y\}$ $\Rightarrow \{\pi\} \left[Ay^2 + By^{\frac{7}{2}} + Cy^5 \right]$ at least one of their terms with the correct power	M1	3.4
	$= \{\pi\} \left[\frac{y^2}{2} + \frac{4k}{7} y^{\frac{7}{2}} + \frac{k^2}{5} y^5 \right]$ $= \{\pi\} \left[\frac{y^2}{2} - \frac{2}{35} y^{\frac{7}{2}} + \frac{1}{500} y^5 \right]$	A1ft	1.1b
	$V = \pi \left(8 - \frac{256}{35} + \frac{256}{125} \right) - (0) + \frac{2}{25}\pi$ $V = \frac{2392}{875}\pi + \frac{2}{25}\pi$	M1	3.4
	$V = \frac{2462\pi}{875} \text{ cm}^3$	A1	2.2b
		(7)	
(c)	E.g. • The equation of the curve may not be a suitable model • The sides of the ornament will not be perfectly smooth • There may be flaws/bubbles within the glass • The corner (ABC) may not be a perfect right angle	B1	3.5b
		(1)	

(d)	Makes an appropriate comment that is consistent with their value for the volume and 9 cm^3. Some evidence of making a comparison and draws a conclusion E.g. a good estimate as 8.84 cm^3 is only 0.16 cm^3 less than 9 cm^3 • A volume between 8.5 and 9.5 is a good model • A volume between 8 and 10 can be either a good or bad model • A volume less than 8 or more than 10 is a bad model, over estimate or underestimate • model volume is less, not enough glass would be ordered so it is a bad model, following a correct answer to (b)	B1ft	3.5a
		(1)	
			(11 marks)

Notes

(a)
M1: Substitutes (0.4, 4) into the equation modelling the curve in an attempt to find the value of *k*
A1: Infers from the data in the model, the value of *k*
(b)
B1: Uses the information given in the model to establish the correct volume of the cylinder
M1: Uses the model and applies $\pi\int x^2 \{dy\}$, d*y* not required and π may appear later in their solution. If they find an expression for x^2 first and then substitutes into the formula score M1 even if an incorrect expansion.
A1ft: Correct expression for the volume generated by the curve with the bracket expanded (follow through their *k* value), d*y* not required and π may appear later in their solution. Indices need to be processed for this mark, may be seen later in the solution.
M1: Attempts to integrate with at least one power raised by 1
A1ft: Correct integration (follow through on their expression for x^2 as long as there are 3 terms). Need not be simplified.
M1: Uses the correct limits and finds the sum of the 2 volumes. Must come from an attempt at $\pi\int_0^4 x^2 \{dy\}$ and an attempt at the volume of the cylinder, condone incorrect formula used as long as it is 3 dimensional not an area.
A1: $\frac{2462\pi}{875}$
Use of calculator scores a maximum of B1M1A0M0A0M1A0 volume = π2.7337…
(c)
B1: States an acceptable limitation of the model, which is the curve but accept flaws/bubbles in the glass. Measurements may not be accurate, or anything related to thickness is B0
(d)
B1ft: Compares the actual volume to their answer to part (b) and makes an assessment of the model with a reason. If using a percentage error then they must use 9 as the true volume.

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